Triple-humped soliton solution for a lattice equation related to the discrete KdV equation

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## LETTER TO THE EDITOR

## Triple-humped solition solution for a lattice equation related to the discrete KdV equation

Kazuaki Narita<br>B1010, 31 Yamada-Nishi 1-Chome, Suita-Shi, Osaka 565, Japan

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#### Abstract

A triple-humped solition solution is found for a lattice equation related to the discrete KdV equation.


We consider the discrete Kdv equation [1] on even lattice sites

$$
\begin{equation*}
\dot{N}_{i}=\left(\tilde{N}_{i-2}-\hat{N}_{i+2}\right)\left(1+\bar{N}_{i}\right)^{2} \tag{1}
\end{equation*}
$$

For equation (1), using the transformation

$$
\begin{equation*}
1+N_{i}=2 /\left[\left(1-u_{i+2}-u_{i+1}\right)^{1 / 3}+\left(1-u_{i}-u_{i-1}\right)^{1 / 3}\right] \tag{2}
\end{equation*}
$$

we find that $u_{i}$ must satisfy the closed equation

$$
\begin{align*}
& \dot{u}_{i}=12\left[\frac{\left(1-u_{i+2}-u_{i+1}\right)^{1 / 3}\left(1-u_{i}-u_{i-1}\right)^{1 / 3}}{\left(1-u_{i+2}-u_{i+1}\right)^{1 / 3}+\left(1-u_{i}-u_{i-1}\right)^{1 / 3}}\right. \\
&\left.\quad-\frac{\left(1-u_{i+1}-u_{i}\right)^{1 / 3}\left(1-u_{i-1}-u_{i-2}\right)^{1 / 3}}{\left(1-u_{i+1}-u_{i}\right)^{1 / 3}+\left(1-u_{i-1}-u_{i-2}\right)^{1 / 3}}\right] . \tag{3}
\end{align*}
$$

We found that equation (3) possesses a soliton solution in the form

$$
\begin{align*}
u_{i}=\tanh (2 k) & \operatorname{sech}^{2}(2 k)\left(3 \tanh (x+k / 2)+\sinh ^{2}(2 k) \tanh ^{3}(x+k / 2)\right) \\
& -\tanh (2 k) \operatorname{sech}^{2}(2 k)\left(3 \tanh (x-k / 2)+\sinh ^{2}(2 k) \tanh ^{3}(x-k / 2)\right)  \tag{4}\\
= & \frac{6 \sinh k \tanh (2 k)}{\cosh (2 x)+\cosh k}-\frac{6 \sinh (2 k) \tanh ^{3}(2 k)}{(\cosh (2 x)+\cosh k)^{2}}+\frac{8 \sinh ^{3} k \tanh ^{3}(2 k)}{(\cosh (2 x)+\cosh k)^{3}} \tag{5}
\end{align*}
$$

in which

$$
\begin{equation*}
x=k i-\sinh (4 k) t+\delta . \tag{6}
\end{equation*}
$$

The plot of equation (5) is shown in figure 1.
The following properties of the soliton solution (5) are obvious. Define $k_{ \pm}$by

$$
\begin{equation*}
k_{ \pm}=\cosh ^{-1} z_{ \pm} \tag{7}
\end{equation*}
$$

where $z_{ \pm}$are two real roots of the equation

$$
\begin{equation*}
4 z^{4}-16 z^{3}+12 z^{2}+1=0 \tag{8}
\end{equation*}
$$



Figure 1. The variation of the soliton profile: (a) $k=0.4$; (b) $k=1$; (c) $k=2$; (d) $k=3$.
Then, the solution has one peak for $|k| \leqslant k_{-}$, two peaks for $k_{-}<|k| \leqslant k_{+}$, and three peaks for $|k|>k_{+}$.

Since equation (3) is directly related to the integrable equation (1) through the transformation (2), it is obvious that the soliton does not lose its identity after a multiple collision. The detail of the collisional process awaits a future analysis.

## Reference

[1] Hirota R 1977 J. Phys. Soc. Japan 431424

