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LETTER TO THE EDITOR

Triple-humped solition solution for a lattice equation related to the discrete KdV equation

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Abstract. A triple-humped solition solution is found for a lattice equation related to the discrete KdV equation.

We consider the discrete KdV equation [1] on even lattice sites

$$\dot{N}_i = (N_{i-2} - N_{i+2})(1 + N_i)^2. \tag{1}$$

For equation (1), using the transformation

$$1 + N_i = 2/[(1 - u_{i+2} - u_{i+1})^{1/3} + (1 - u_i - u_{i-1})^{1/3}] \tag{2}$$

we find that u_i must satisfy the closed equation

$$\dot{u}_i = 12\left[\frac{(1-u_{i+2}-u_{i+1})^{1/3}(1-u_i-u_{i-1})^{1/3}}{(1-u_{i+2}-u_{i+1})^{1/3}+(1-u_i-u_{i-1})^{1/3}} - \frac{(1-u_{i+1}-u_i)^{1/3}(1-u_{i-1}-u_{i-2})^{1/3}}{(1-u_{i+1}-u_i)^{1/3}+(1-u_{i-1}-u_{i-2})^{1/3}}\right]. \tag{3}$$

We found that equation (3) possesses a soliton solution in the form

$$u_i = \tanh(2k)\,\mathrm{sech}^2(2k)(3\tanh(x+k/2) + \sinh^2(2k)\tanh^3(x+k/2)) - \tanh(2k)\,\mathrm{sech}^2(2k)(3\tanh(x-k/2) + \sinh^2(2k)\tanh^3(x-k/2)) \tag{4}$$

$$= \frac{6\sinh k\tanh(2k)}{\cosh(2x)+\cosh k} - \frac{6\sinh(2k)\tanh^3(2k)}{(\cosh(2x)+\cosh k)^2} + \frac{8\sinh^3 k\tanh^3(2k)}{(\cosh(2x)+\cosh k)^3} \tag{5}$$

in which

$$x = ki - \sinh(4k)t + \delta. \tag{6}$$

The plot of equation (5) is shown in figure 1.

The following properties of the soliton solution (5) are obvious. Define $k_\pm$ by

$$k_\pm = \cosh^{-1} z_\pm \tag{7}$$

where $z_\pm$ are two real roots of the equation

$$4z^4 - 16z^3 + 12z^2 + 1 = 0. \tag{8}$$

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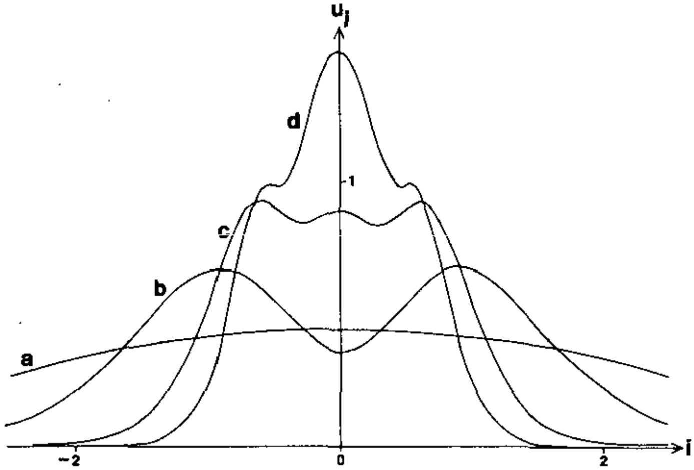

Figure 1. The variation of the soliton profile: (a) $k=0.4$; (b) $k=1$; (c) $k=2$; (d) $k=3$.

Then, the solution has one peak for $|k| \leqslant k_-$, two peaks for $k_- < |k| \leqslant k_+$, and three peaks for $|k| > k_+$.

Since equation (3) is directly related to the integrable equation (1) through the transformation (2), it is obvious that the soliton does not lose its identity after a multiple collision. The detail of the collisional process awaits a future analysis.

Reference

[1] Hirota R 1977 *J. Phys. Soc. Japan* **43** 1424