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LETTER TO THE EDITOR

Triple-humped solition solution for a lattice equation related to the discrete Kav equation

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Abstract. A triple-humped solition solution is found for a lattice equation related to the discrete KdV equation.

We consider the discrete Kav equation [1] on even lattice sites

$$\dot{N}_i = (N_{i-2} - N_{i+2})(1 + N_i)^2.$$
⁽¹⁾

For equation (1), using the transformation

$$1 + N_i = 2/[(1 - u_{i+2} - u_{i+1})^{1/3} + (1 - u_i - u_{i-1})^{1/3}]$$
⁽²⁾

we find that u_i must satisfy the closed equation

$$\dot{u}_{i} = 12 \left[\frac{(1 - u_{i+2} - u_{i+1})^{1/3} (1 - u_{i} - u_{i-1})^{1/3}}{(1 - u_{i+2} - u_{i+1})^{1/3} + (1 - u_{i} - u_{i-1})^{1/3}} - \frac{(1 - u_{i+1} - u_{i})^{1/3} (1 - u_{i-1} - u_{i-2})^{1/3}}{(1 - u_{i+1} - u_{i})^{1/3} + (1 - u_{i-1} - u_{i-2})^{1/3}} \right].$$
(3)

We found that equation (3) possesses a soliton solution in the form

$$u_i = \tanh(2k) \operatorname{sech}^2(2k)(3 \tanh(x+k/2) + \sinh^2(2k) \tanh^3(x+k/2))$$

$$-\tanh(2k) \operatorname{sech}^{2}(2k)(3 \tanh(x-k/2)+\sinh^{2}(2k) \tanh^{3}(x-k/2))$$
(4)

$$= \frac{6\sinh k \tanh(2k)}{\cosh(2x) + \cosh k} - \frac{6\sinh(2k) \tanh^{3}(2k)}{(\cosh(2x) + \cosh k)^{2}} + \frac{8\sinh^{3} k \tanh^{3}(2k)}{(\cosh(2x) + \cosh k)^{3}}$$
(5)

in which

$$x = ki - \sinh(4k)t + \delta. \tag{6}$$

The plot of equation (5) is shown in figure 1.

The following properties of the soliton solution (5) are obvious. Define k_{\pm} by

$$k_{\pm} = \cosh^{-1} z_{\pm} \tag{7}$$

where z_{\pm} are two real roots of the equation

$$4z^4 - 16z^3 + 12z^2 + 1 = 0. ag{8}$$

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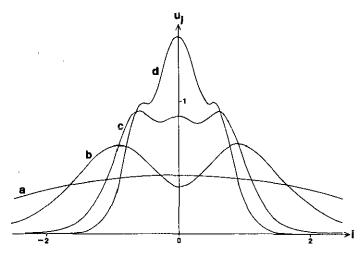


Figure 1. The variation of the soliton profile: (a) k = 0.4; (b) k = 1; (c) k = 2; (d) k = 3.

Then, the solution has one peak for $|k| \le k_-$, two peaks for $k_- < |k| \le k_+$, and three peaks for $|k| > k_+$.

Since equation (3) is directly related to the integrable equation (1) through the transformation (2), it is obvious that the soliton does not lose its identity after a multiple collision. The detail of the collisional process awaits a future analysis.

Reference

[1] Hirota R 1977 J. Phys. Soc. Japan 43 1424